PARAMETRIC MODELLING OF BURST-TYPE SIGNALS

by

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PARAMETRIC MODELLING OF BURST-TYPE SIGNALS

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by

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CERTIFICATE

It is certified that the work contained in the thesis titled **Parametric modelling of Burst-type signals** by Rajneesh Kumar Sharma has been carried out under my supervision and that this work has not been submitted elsewhere for a degree

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ABSTRACT

The complex amplitude modulated signal model with exponential functions as modulating signals as a representation of non stationary burst-type signals exhibiting certain pseudoperiodicity is studied The estimation of model parameters is carried out using time correlations The model is fitted on a noise corrupted computer synthesised marine type signal The suitability of the model for a real signal is also studied by fitting a real electrocardiograph signal The study demonstrates the suitability of the model and elaborates the approach for estimation of parameters

CHAPTER 1

Introduction

Many naturally occurring signals are non-stationary in nature for example the electrocardiograph (ECG) signal is a non-stationary signal (GRENIER 1983) The electrocardiograph signal depicts the electrical activity of the heart The cycle of these electrical impulses is repeated with a frequency which is given by the rate of heart beat of an individual There are various lead configurations for recording the ECG signal Whatever the lead configuration used the ECG signal is characterised by distinctive burst type features corresponding to the various parts of the heart i e QRS P and T waves (GUYTON 1986) These burst type features are highly restricted in time i e , they are localised over a small time interval In addition the ECG signal is marked by its periodicity The burst type QRS feature of an ECG signal contributes localised high frequency components making the ECG signal a distinctly non-stationary signal (WALDO and CHITRAPA 1991) Although this feature of the QRS wave has helped detection of the wave by filtering/linear prediction (FRIESEN et al , 1990), it makes the modelling of the

ECG signal difficult

The marine type signal is also characterised by localised high frequency components The signal exhibits sudden oscillating bursts followed by flat features Although this signal does not exhibit periodicity property like the ECG signal it can still be treated in the manner of the ECG signal by imparting periodicity to it by considering a particular segment of the signal and then letting it repeat in the subsequent periods

The time dependant Autoregressive (AR)/auto regressive moving average (ARMA) model is the representative of the general class of non-stationary signals (GRENIER 1983) As such the model can also be used for the ECG signal However it will be worthwhile to know how the special burst features and the periodic nature of the ECG signal restrict the general time dependant AR/ARMA model

It has been shown that the amplitude modulated (AM) sinosoidal signal model can have the periodicity property and the model can exhibit burst type features very well when the modulating signal is an exponential function (MUKHOPADHYAY and SIRCAR 1996)

It is seen that a total of four AM sinosoids are needed to model separately each feature of the signal It is proposed that in general one sinosoid is required to represent each burst of a burst type signal This model has been ascertained to represent the ECG signal fairly accurately using analysis by synthesis approach (MUKHOPADHYAY and SIRCAR 1996) In this thesis the estimation of the model parameters is attempted A study of these parameters brings out the following facts The modulating signal consists of a very pronounced burst limited to a small fraction of the total time period of the signal The modulating signal is strictly greater than zero and it s magnitude beyond the burst feature is negligible as compared to the burst magnitude itself The carrier signal has a very small amplitude as compared to the peak of the signal it is being used to model The magnitude of the burst thus largely depends upon the modulating signal magnitude

The above features of the burst type signals in general and those of ECG signal in particular are exploited to achieve easy separation of the waves, representing the signal in this thesis The waves so separated are seen to be contributing nothing beyond the burst type features to the overall signal

It is shown in this thesis that the AM sinosoidal signal model with exponential modulating signals can be used effectively to represent burst type signals and the model parameters estimated The approach gives a satisfactory performance in the presence of noise

Organisation of the Thesis

The present thesis has been organised into 5 Chapters Chapter 1 deals with the introduction and organisation of the thesis Chapter 2, introduces the model Chapter 3 shows the approach applied for estimation of the model parameters Chapter 4 includes two simulation studies one on a synthesised marine type signal and the other on a real ECG signal Chapter 5 indicates the scope for further work besides concluding remarks

CHAPTER 2

AM Signal Model with Exponential Modulating Signal

2 1 The Model Equation

Let the discrete time random process x(n) represent the signal where x(n) is given by

$$x(n) = \sum_{i=1}^{M} A_i \exp\left\{b_i \left[1 - \cos\left(\alpha n + c_i\right)\right]\right\} \exp\left\{j\left[\theta_i n + \phi_i)\right]\right\} \qquad (2\ 1)$$

where

A_i = amplitude of the carrier wave

b_i = part of the modulating factor which is given by $\exp\left[- b_i \cos\left(\alpha n + c_i\right)\right]$

c_i = phase of the modulating frequency

θ_i = carrier frequency

ϕ_i = uniformly distributed random variable with distribution $U\ [0\ \ 2\pi]$

α = modulating frequency

M = number of waves

n = time index

Note that α and θ_i are discrete angular frequencies where the factor 2π and the sampling interval have been absorbed into them To cater for the pseudoperiodic nature of the ECG signal θ_i are constrained to be integer multiples of α i e

$$\theta_i = k_i \alpha \qquad (2\ 2)$$

where k_i is an integer

2 2 Model Parameters and the Wave Shapes

It is pertinent to see in some detail how these parameters in the model effect the shape of the waves they represent As seen from (2 1) the modulating signal is given by

$$\exp\left\{b_i\left[1 - \cos\ (\alpha n + c_i)\right]\right\} \qquad (2\ 3)$$

There is one such modulating signal with different values of the parameters for each wave, except for the modulating frequency which is the same for all the waves and is a function of the periodicity of the signal The signal is imparted the periodicity property due to the above in conjunction with (2 2) where the carrier signal frequency is an integer multiple of the modulating frequency The modulating signal is strictly positive with a very sudden and well pronounced peak centered around N_p given by

$$N_p = \frac{\pi - c_i}{\alpha} \qquad (2\ 4)$$

To allow for well demarcated bursts the amplitude of the carrier signal will be very insignificant as compared to the peak of the signal The prominence to the peak is imparted by a high value of b_i In addition to deciding the location of the peak the factor c_i provides with a freedom in the choice of origin while processing the signal

The modulating signal of (2 3) is being multiplied by a carrier signal,

$$A_i \exp \left\{ j \left[\theta_i n + \phi_i) \right] \right\} \qquad (2\ 5)$$

A low value of A_i ensures that the particular i th wave does not contribute to the overall signal being modelled, beyond the limits of its burst feature The phase ϕ_i decides the polarity of the peaks In addition it can cater for a slight skewness of these humps particularly if the modelled signal happens to be symmetrically placed about the line $x = 0$

2 3 Frequency Analysis of the Model

The frequency parameters of the model can be extracted using Fourier analysis after processing the signal as follows Starting from (2 1) we proceed to find the correlations, Rx1(n) and Rx2(n) as defined below,

$$Rx1(n) = E\ \{x(n)\quad x^*(n)\} \qquad (2\ 6)$$

$$Rx2(n) = E\ \{x(n)\quad x^*(-n)\} \qquad (2\ 7)$$

where E stands for the expectation operator and $x^*(n)$ denotes the complex conjugate of x(n)

Substituting the values of x(n) and $x^*(n)$ from (2 1) and evaluating the right hand side of (2 6) we get

$$Rx1(n) = \sum_{i=1}^{M} A_i^2 \exp\left\{2b_i\left[1 - \cos\ (\alpha n + c_i)\right]\right\} \qquad (2\ 8)$$

where the following result of expectation is used

$$\begin{aligned} E\ \{\exp j\ (\phi_i - \phi_l)\} &= 1 \qquad \text{for } \phi_i = \phi_l \\ &= 0 \qquad \text{for } \phi_i \neq \phi_l \end{aligned}$$

Similarly Rx2(n) evaluates to

$$Rx2(n) = \sum_{i=1}^{M} A_i^2 \exp\left\{2b_i\left[1 - \cos\ (\alpha n + c_i)\right]\right\} \exp\left\{ j2\theta_i n\right\} \qquad (2\ 9)$$

It can be seen from (2 8) that Rx1(n) is nothing but the square of the modulating signal (2 3) multiplied by the square of the amplitude It is free of the carrier frequency term and is always positive

As previously brought out the burst type nature of the signal characterised by sudden sharp rise followed by relatively flat features allows for the treatment of waves as existing only during the presence of the burst For example let a signal sequence x(n), exhibit sharp sudden bursts from n=10 to n=20 and again from n=45 to n=60 The signal, say repeats after n=60 samples The burst type features allow us to treat the signal as composed of two waves of our model each corresponding to one of the bursts The first wave sequence $x_1(n) = x(n)$ for n=10 to n=20 and it is equal to zero for n=1 to 9 and n=21 to 60 Similarly $x_2(n)$ is equal to x(n) during the presence of the burst from n=45 to n=60 This separation can be done from the signal to be modelled itself However Rx1(n) as in (2 8) exhibits sharp and well demarcated bursts and can also be used for the same

It will be easier to deal with individual waves as far as estimation of parameters is concerned than to use their sum Hence we deal with each wave separately as under after determining the total number of such waves required to model the signal

An individual wave in the model is represented as

$$x_1(n) = A_1 \exp\left\{2b_1\left[1 - \cos(\alpha n + c_1)\right]\right\} \exp\left\{ j \ (\theta_1 n+\phi_1)\right\} \tag{2 10}$$

Similarly the waves $x_2(n)$, $x_3(n)$ $x_M(n)$ are formed for a signal needing M waves to model it The number of M waves required is given by the number of peaks of Rx1(n) (2 8) The values of $x_1(n)$ i = 1 to M are equated to those of x(n) within the time indices corresponding to the i th burst of Rx1(n)

From here on we will deal with one such wave say that corresponding to i=1 After forming $x_1(n)$ from x(n) we find the correlations corresponding to (2 8) and (2 9) as refered to $x_1(n)$

$$Rx11(n) = E\left[x_1(n) \quad x_1^*(n)\right] \qquad (2\ 11)$$

$$Rx21(n) = E\left[x_1(n) \quad x_1^*(-n)\right] \qquad (2\ 12)$$

Using the values of $x_1(n)$ and $x_1^*(n)$ the above two equations evaluate to

$$Rx11(n) = A_1^2 \exp\left\{2b_1\left[1 - \cos\ (\alpha n + c_1)\right]\right\} \qquad (2\ 13)$$

$$Rx21(n) = A_1^2 \exp\left\{2b_1\left[1 - \cos\ (\alpha n + c_1)\right]\right\} \exp\left\{\ j2\theta_1 n\right\}$$

(2 14)

Further expanding Rx11(n) from 2 13 shows that it is a sum of sinosoids at frequencies that are integer multiples of α and a large DC term

Writing (2 13) as under

$$Rx11(n) = B \exp\left[D \cos\ (\alpha n + c_1)\right] \qquad (2\ 15)$$

where $D = -2b_1$

$B = A_1^2 \exp\,[-D]$

we have

$$Rx11(n) = B\left[1+D\cos(\alpha n + c_1) + \frac{D^2}{2!}\cos^2(\alpha n + c_1) + \frac{D^3}{3!}\cos^3(\alpha n + c_1) + \quad\right] \qquad (2\ 16)$$

(2 16) shows that Rx11(n) is composed of sinosoids at frequencies α 2α, 3α These sinosoids have amplitudes that decrease with increasing frequency In addition it has a DC component larger than the amplitudes of any of the sinosoid

The Fourier transform of 2 16 will thus have a global maximum at f=0 besides local maxima at $f = \pm n\alpha$ where n = 1 2 3

It is seen from (2 13) and (2 14) that Rx11(n) and Rx21(n) are related as follows

$$Rx21(n) = Rx11(n)\ \exp\{j\ 2\theta_1 n\} \qquad (2\ 17)$$

Hence Fourier transform of (2 14) will be the same as that of (2 13) shifted by $f = 2\theta_1$ i e with a global maxima at $f = 2\theta_1$

For taking the Fourier transforms of the discrete sequences Rx11(n) and Rx21(n) which are real and complex respectively suitable fast Fourier transform (FFT) algorithms can be used to take an L point transform where L >> N N being the number of points in the sequence

2 4 Concluding Remarks

From the foregoing it is seen that the time correlations of the signals as defined in (2 11) and (2 12) can be used to estimate the frequencies of the model The modulating frequencies will come out to be the same for all the waves Minor corrections to the θ values will be needed to compensate for the effects of noise These corrections will involve ensuring that the θ values are integer multiples of the modulating frequency

It is also mentioned that any higher order moment of the type used here will not yield any additional information about the signal This is suggested by the forms of (2 8) and (2 9)

CHAPTER 3

Model Fitting and Parameter Estimation

3 1 Separating the Waves

Starting with the sampled sequence x(n) n = 0 1 N-1 where N = number of samples per period of the signal we form Rx1(n) as under,

$$Rx1(n) = x(n)\ x^{*}(n) \qquad (3\ 1)$$

Note that the expectation operator as in (2 6) has been dropped in (3 1) This is because we are actually dealing with a non-stationary signal where the notion of ergodicity does not apply The highly localised and pronounced bursts in the sequence x(n) will give rise to Rx1(n) which will have well demarcated sharp bursts followed by near zero amplitudes The bursts will be symmetrical about the peaks In general the number of such peaks will be equal to the number of burst type features in the signal to be modelled The M waves are thus separated from the signal x(n), with the value of the samples of individual M waves equated to zero beyond the duration of the burst they are meant to represent This approximation is valid only for signals that posses these well defined burst type features

3 2 Estimation of Frequencies

To outline the approach for determination of the frequencies of the model, we will proceed with one of the M waves say the first such wave $x_1 n$ We form Rx11(n) and Rx21(n) as under

$$Rx11(n) = x_1(n) \quad x_1^*(n) \qquad (3\ 2)$$

$$Rx21(n) = x_1(n) \quad x_1^*(-n) \qquad (3\ 3)$$

The expectation operator is of course again dropped The Fourier transform (DFT) of (3 3) will have a peak at $f = 2\theta_1$ The actual discrete angular frequency is given by multiplying $2\theta_1$ by π to get the frequency equal to θ_1 The value of α can be found by taking the discrete Fourier transform of the complete signal itself It will have local maxima separated by α The required correction is applied to the value of θ_1 to confirm that θ_1 is an integer multiple of α The above procedure is applied to all the M waves to get M values for the carrier frequencies to be used in the model

3 3 Estimation of Other Parameters

The approach is demonstrated below for one of the waves, say the first waves We make use of Rx11(n) for estimation of the parameters which is shown below

$$\text{Rx11}(n) = A_1^2 \exp\left\{2b_1\left[1 - \cos(\alpha n + c_1)\right]\right\} \qquad (3\ 4)$$

The problem of estimation of parameters in (3 4) is a highly non-linear one However it can be converted to a linear problem by first evaluating the value of c_1 The evaluation of c_1 as also that of other c_i $i = 2$ to M can be done making use of (3 1) which is shown below

$$\text{Rx1}(n) = \sum_{i=1}^{M} A_i^2 \exp\left\{2b_i\left[1 - \cos(\alpha n + c_i)\right]\right\} \qquad (3\ 5)$$

this will have well demarcated and conspicuous peaks at n_i given by

$$n_i = \frac{\pi - c_i}{\alpha} \qquad (3\ 6)$$

Since the peaks occur when $\cos(\alpha n + c_i)$ is equal to -1

Noting the values of n_i corresponding to the peaks of (3 5) we get c_i as follows

$$c_1 = \pi - \alpha n_1 \qquad (3\ 7)$$

Any negative phase is then absorbed if it turns out in (3 7)

Now we use the value of c_1 as found out from above to reformulate the problem of (3 4)

$$X(n_1) = \ln A_1 + b\ Y(n_1)$$

$$X(n_1+1) = \ln A_1 + b\ Y(n_1+1)$$

$$X(n_2) = \ln A_1 + b\ Y(n_2) \qquad (3\ 8)$$

where $$X(n) = \frac{\ln Rx11(n)}{2}$$

$$Y(n) = 1 - \cos\ (\alpha n + c_1)$$

and n_1 and n_2 are the extremeties of the burst corresponding to the first wave

The problem in (3 8) is a linear problem in two unknowns which is highly over determined with $n_2 - n_1 >> 2$ This system of equations is solved in the least squares sense to get the estimates of A_1 and b_1 Solving (3 8) with just the requisite number of equations also gives accurate solutions in the absence of noise The least squares approach makes the solutions robust to cater for the noise

A similar procedure is followed for the other M-1 waves   To estimate the values of ϕ_1 a direct evaluation from the signal can be resorted to   However it is found that to get accurate results one can resort to fitting the estimated signal using the estimated values over the actual signal   The fitting involves sliding the estimated signal over the actual one and then using the ϕ_1 corresponding to the minimum error   A criterion like the minimum least square error criterion can be used   The fitting is done on the real signal which is derived from the given sequence as follows

$$y(n) = \frac{x(n) + x^{*}(n)}{2} \qquad (3\ 9)$$

where y(n) is the real signal

3 4 Concluding Remarks

It is stressed that it is only while estimating the parameters of the signal, that we resort to approximating the individual M waves by the particular burst in the signal that they are employed to model while keeping the rest of the wave equal to zero   But while regenerating the estimated signal using the estimated parameters we allow the waves to exist during the entire period of the signal   That is having estimated the parameters we reconstruct the signal using the initial model equation as in (2 1)

The procedure developed provides a robust estimation technique because of two reasons Firstly for the estimation we are using the second moment, which smooths out the noise to a large extent Secondly while estimating the crucial parameters A_1 and b_1, we employ the least squares approach in solving the overdetermined linear matrix problem However the least squares approach leads to errors in the vicinity of a very extreme feature affecting one or two points only This can be circumvented using appropriate weight functions to provide for additional weight to the equations in the vicinity of n corresponding to the extreme feature

CHAPTER 4

Simulation Study

4 1 General

The efficacy of the model is demonstrated by two simulation studies in this chapter In the first case we fit the model on a synthesised complex marine type signal In the second case we use a natural real ECG signal of a normal patient The effect of noise is studied by adding computer synthesised noise in the first case

4 2 Example 1 Synthesised Signal

A complex marine type signal consisting of M = 3 amplitude modulated waves is considered here The total number of samples being considered are 256

The parameters chosen are

$A_1 = 1 \times 10^{-10}$ $b_1 = 15\ 0$ $c_1 = 2\ 0$ $\theta_1 = 3681555$ $\phi_1 = 3\ 0$

$A_2 = 1 \times 10^{-10}$ $b_2 = 14\ 8$ $c_2 = 0\ 0$ $\theta_2 = 490874$ $\phi_2 = 2\ 0$

$A_3 = 1 \times 10^{-10}$ $b_3 = 15\ 0$ $c_3 = 4\ 0$ $\theta_3 = 245437$ $\phi_3 = 3\ 5$

The α value is chosen as 0245437

First the product $Rx1(n) = x(n)\ x^*(n)$ is computed This product is nothing but the square of the three modulating signals multiplied by the square of the amplitudes of the carrier waves This sequence Rx1(n) shows three sharp bursts corresponding to the three waves required to model the signal In fact Rx1(n) comprises of the cross terms between different waves also, but due to the burst nature these cross terms are negligible This fact is shown by the plot of Rx1(n) and that of the square of modulating signal multiplied by the respective carrier wave amplitudes Fig 4 1 shows that Rx1(n) is infact equal to the following term

$$Rx1(n) = \sum_{i=1}^{M} A_i^2 \exp\left\{2b_i\left[1 - \cos\ (\alpha n + c_i)\right]\right\} \qquad (4\ 1)$$

After demarcating the signal into the three waves required to model it form (4 1) we form the individual correlations of the three waves as shown earlier at (3 2) and (3 3) The DFT plots of these correlations will give the values of θ_1 θ_2 and θ_3 The shifting of the discrete Fourier transform of Rx11(n) by $2\theta_1$ when we take the Fourier transform of Rx21(n) is shown at Fig 4 2 Fig 4 3 and 4 4 show the similar shifting in case of waves corresponding to θ_2 and θ_3 For estimating α we make use of the discrete Fourier transform of the actual signal itself The plot showing well demarcated and regularly spaced peaks at an interval of 'α is shown at Fig 4 5

We next compute the values of c_1 c_2 and c_3 from (4 1) in accordance with (3 6) and (3 7) The matrix problem in (3 8) is solved next for each of the three waves separately in the least squares sense No weight functions are required in this particular case We next fit the signal estimated using the above estimates over the actual signal and estimate ϕ_1 ϕ_2 and ϕ_3

4 3 Estimation in the Presence of Noise

The effect of noise on the estimation procedure is studied by adding computer synthesised Gaussian zero mean additive noise setting the signal to noise ratio (SNR) at 30 db 20 db and 10 db respectively The plots showing the DFT operation for estimating θ_1 are shown at Fig 4 6 4 7 and 4 8 For SNR equal to 30, 20 and 10 db respectively

The actual signal along with the one estimated under no-noise condition is shown at Fig 4 9 Fig 4 10 shows the estimated signal when the SNR is 30 and 20 db respectively The actual values of the parameters and those estimated under different noise conditions is tabulated in Table 4 1

From Fig 4 8 it is seen that though noise is superimposed on the DFT plot, the peak and its location is still unaffected by noise even with the SNR = 10 db

The comparison of actual signal and those estimated shows that the procedure developed is quiet robust

4 4 Example 2 A Real ECG Signal

In this example the suitability of the developed model for a natural real ECG signal is studied A lead II configuration ECG signal of a normal patient is employed for the purpose The signal has been sampled at 250 Hz The natural ECG signal is a highly noisy signal The signal before being processed is averaged over a few periods to smooth out the noise to some extent The highly distinctive peaks of the QRS complex are used as a reference point while carrying out the averaging operation

We consider 166 samples of the signal corresponding to one period of the ECG signal being considered This information is again available from the repetition frequency of the highly distinctive QRS wave s peak

It is seen that the procedure developed does not apply straightaway in the case of a real signal In fact the expectations as in (2 8) and (2 9) turn out to be equal to zero This is because the procedure outlined previously considers the

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presence of one complex signal at one time Here in the case of a real ECG signal we are in fact dealing with two complex signals at the same time whose sum gives the real ECG signal This fact necessitates some preliminary processing on the real ECG signal to get it into a form suitable for processing using the given procedure

For this purpose we convert the real ECG signal into its equivalent complex signal as follows

We start with the given real ECG signal which can be written as

$$y(n) = \sum_{i=1}^{M} A_i \exp\left\{2b_i\left[1 - \cos(\alpha n + c_i)\right]\right\} \cos(\theta_i n + \phi_i) \qquad (4\ 2)$$

The highly localised bursts of the ECG signal allow us to separate it out into its constituent waves needed to model it A total of four waves are required since the ECG signal shows four distinct burst features Thus we form the four signals $y_i(n)$ $i = 1$ to 4 $n = 0, N-1$ This is done by seeing the distinctive features of the signal under consideration The demarcation coincides with the zero crossings of the signal The same zero crossing information is used to get the trial estimates of θ and ϕ for all the four waves It is seen that a bias of -400 when added to the signal makes the estimation more accurate This is because by adding this bias the T wave which is a little gradual and extends over a longer duration gets symmetrically placed about the line y=0

Using (4 2) in conjunction with the trial estimates of θ and ϕ along with the information of time indices comprising the four waves we form the complex signal as follows

$$\frac{y(n)}{\cos\,(\theta_i n + \phi_i)} = A_i \exp\left\{b_i\left[1 - \cos\,(\alpha n + c_i)\right]\right\} \qquad (4\ 3)$$

with n over the appropriate range for particular i Multiplying the RHS of (4 3) by $\sqrt{1 - \cos^2(\theta_i n + \phi_i)}$ gives the imaginary part which when added to the actual signal gives its equivalent complex signal This procedure is repeated for all the four waves

The division at (4 3) poses some problem in the vicinity of the zero crossings This is because the zero crossings are occurring at $\cos\,(\theta_i n + \phi_i) = 0$ Hence the division at 4 3 leads to disproportionate values of the RHS of 4 3 leading to a large imaginary part of the complex signal being generated This problem is overcome by interpolating the values near zero crossings using the values slightly away from it The real and imaginary parts of the complex signal corresponding to the QRS wave are shown at Fig 4 11

Having formed the complex signal we proceed with the procedure as developed earlier Rx1(n) corresponding to the QRS wave is shown at Fig 4 12 The values of θ, corresponding to different waves are confirmed to be the same as the trial estimates The value of α can be estimated either by taking the DFT of the complex signal or by taking the DFT of $Rx2(n)$ corresponding to any of the waves The DFT plot for $Rx2_{qrs}(n)$ is at Fig 4 13 It is seen from this figure that $Rx2_{qrs}(n)$ has a global maximum at frequency equal to twice the frequency of the carrier wave corresponding to QRS wave In addition it has local maxima separated by $f = \alpha$

Care must be taken while solving (3 8) In the ECG signal the QRS complex exhibits a very sharp burst The least squares approach though caters for the effects of noise tends to overlook the very extreme values especially if they happen to be confined to just one or two points Suitable weights will be needed to be attached to the equations at these extreme points while calculating A_1 and b_1 s It is seen that a weight of 1 415 and 1 35 is needed to be attached to the equation dealing with the highest points of QRS complex and the P_2 wave respectively

The signal is then regenerated using the values estimated and the values of ϕ_1 are confirmed to be same as the trial estimates The DC bias of +400 is added back into the signal to compensate for the earlier bias of -400 The actual ECG signal along with the estimated one is shown at Fig 4 14

It is relevant to study the closeness of the estimated signal to the actual one The SNR gives a good measure of the closeness of the two signals where SNR is defined as

$$\mathrm{SNR} = 10 \log \left[\frac{\sum_{i=1}^{N} [x_{org}(i)]^2}{\sum_{i=1}^{N} [x_{org}(i) - x_{syn}(i)]^2} \right]$$

where $x_{org}(i)$ is the original ECG signal

$x_{syn}(i)$ is the estimated signal,

N = number of samples in one period

the SNR for the Fig 4 11 comes out to 16 38 db

The close resemblance of the two signals at Fig 4 11 and the SNR value suggests that the procedure developed can effectively represent the ECG signal in a parametric form

Table 4 1

ACTUAL SIGNAL PARAMETERS

α = 0245437 NO OF SAMPLES N = 256

	A	b	θ	c	ϕ
1	1×10^{-10}	15 0	3681555	2 0	3 0
2	1×10^{-10}	14 8	490874	0 0	2 0
3	1×10^{-10}	15 0	245437	4 0	3 5

ESTIMATED PARAMETERS WITHOUT NOISE

α = 02454

1	$9\ 48132 \times 10^{-11}$	15 0276	3681554	1 98804	3 0
2	$1\ 22466 \times 10^{-10}$	14 6921	4908739	0 0	2 0
3	$1\ 01752 \times 10^{-10}$	14 9887	2454369	4 0006	3 5

ESTIMATED PARAMETERS SNR = 30DB

α = 02454

1	$8\ 94566 \times 10^{-11}$	15 05	3681554	2 01258	3 0
2	$1\ 25937 \times 10^{-10}$	14 6798	4908739	0 0	2 0
3	$1\ 04784 \times 10^{-10}$	14 9743	2454369	4 00062	3 5

ESTIMATED PARAMETERS SNR = 20DB

α = 02454

1	$1\ 20012 \times 10^{-10}$	14 9023	3681554	1 9635	3 0
2	$1\ 17015 \times 10^{-10}$	14 5206	4878059	0 0	2 0
3	$1\ 3214 \times 10^{-10}$	14 8502	2546408	4 02517	3 5

Fig 4 1 RX1(n) vs sq of mod sigs

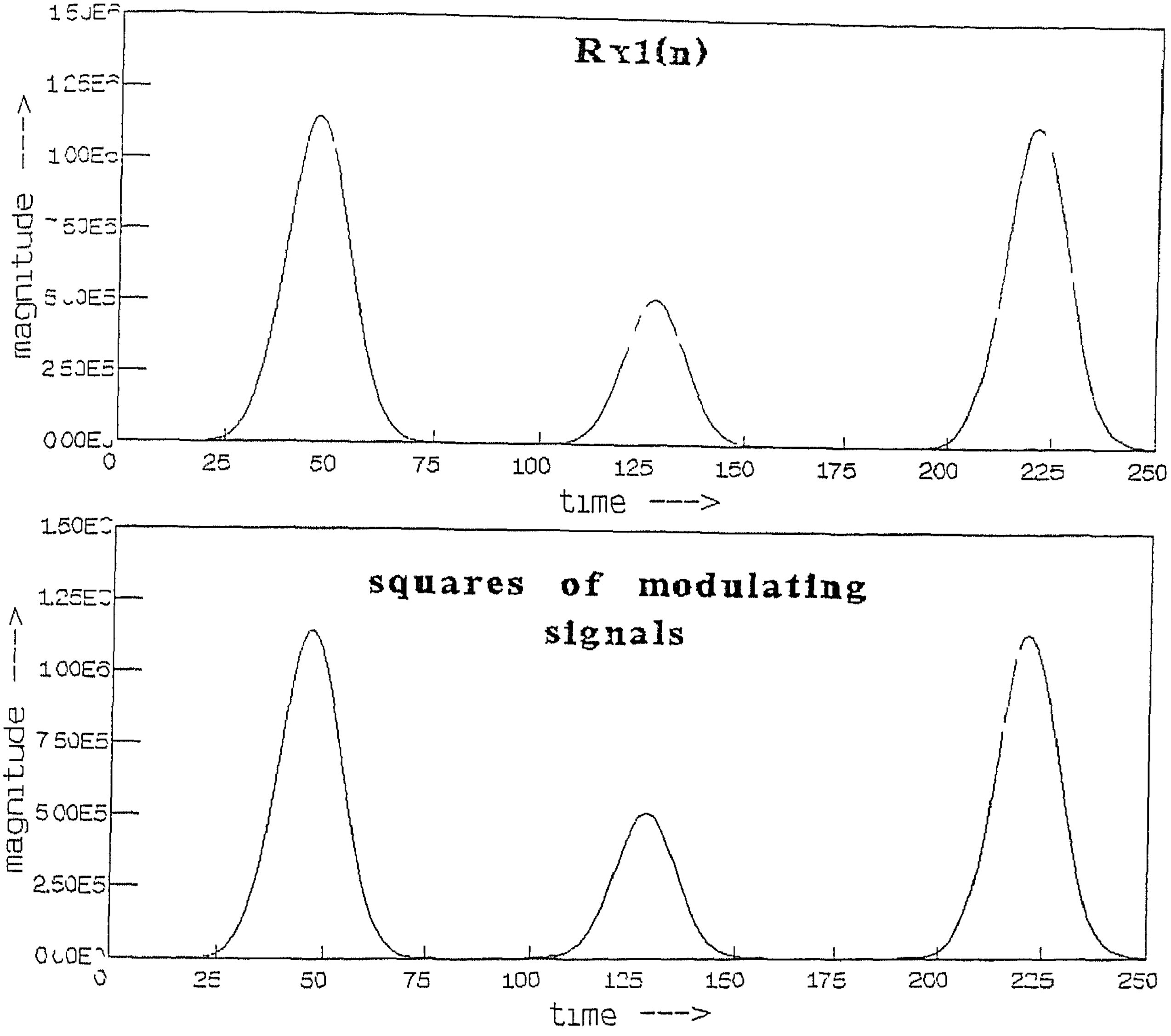
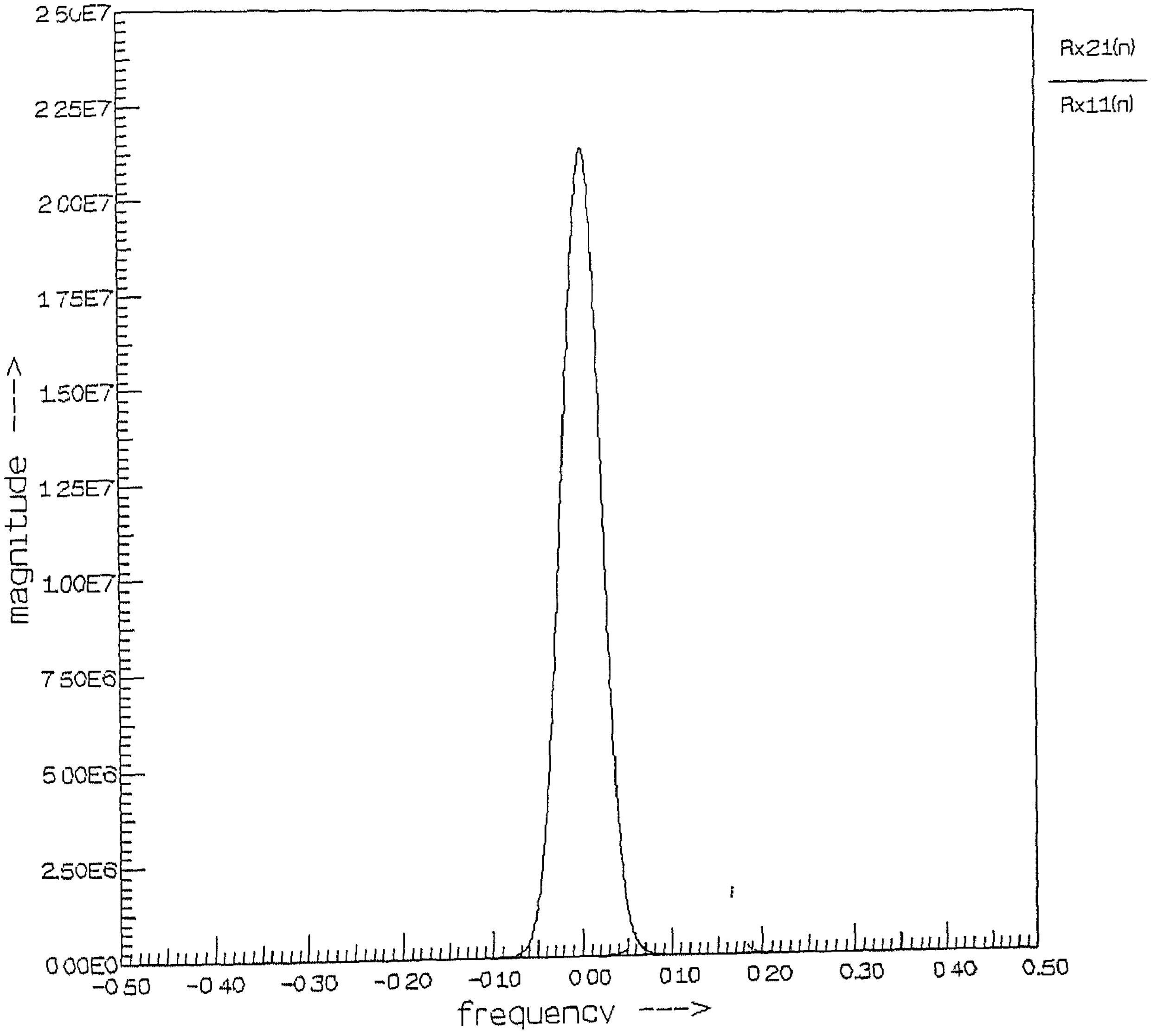


Fig 4 2

Est of theta 1



Us

Fig 43

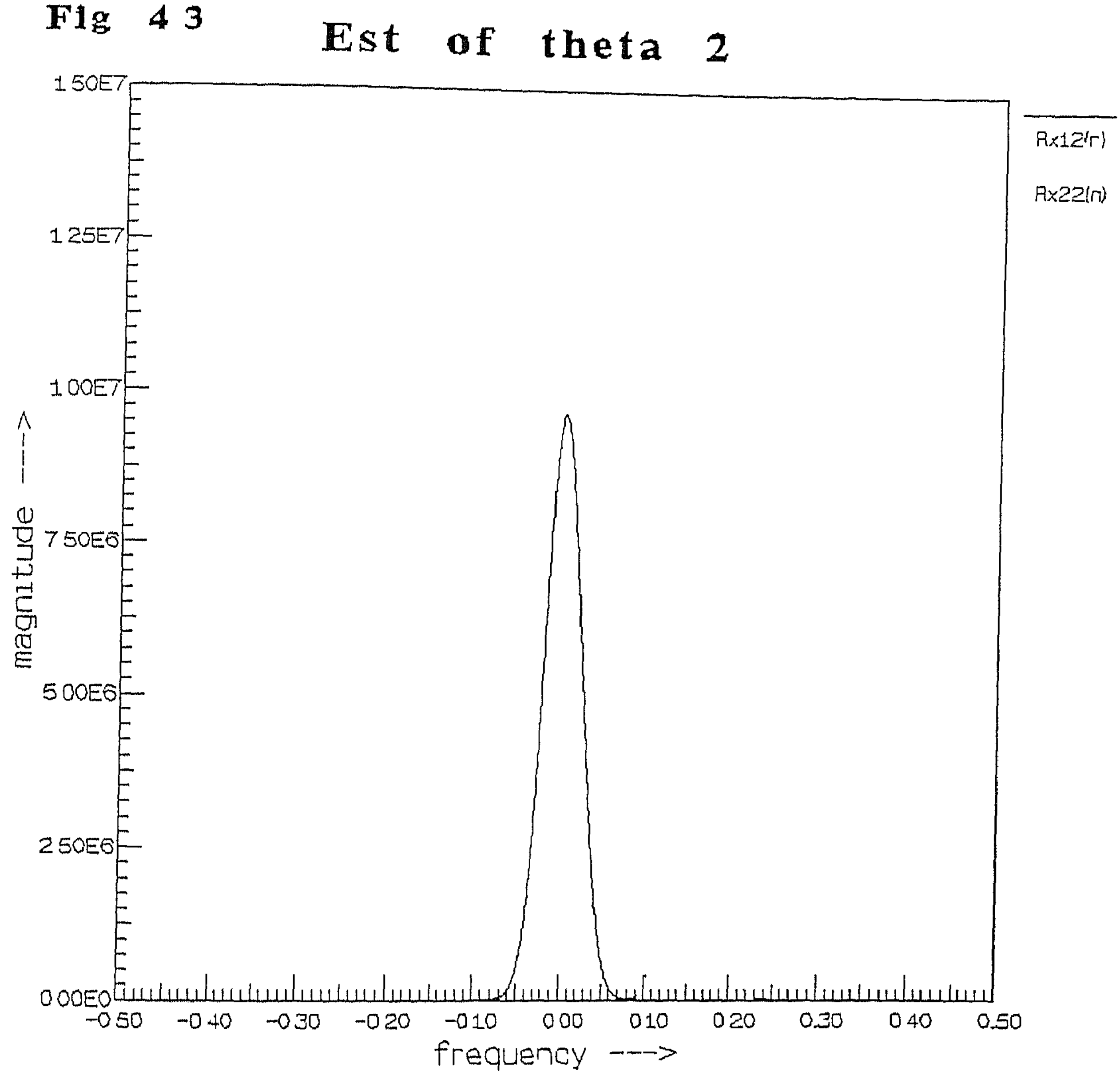

Est of theta 2
1.50E7
1.25E7
1.00E7
7.50E6
5.00E6
2.50E6
0.00E0
magnitude --->
-0.50
-0.40
-0.30
-0.20
-0.10
0.00
0.10
0.20
0.30
0.40
0.50
frequency --->
Rx12(n)
Rx22(n)

Fig 4 4 **Est of theta 3**

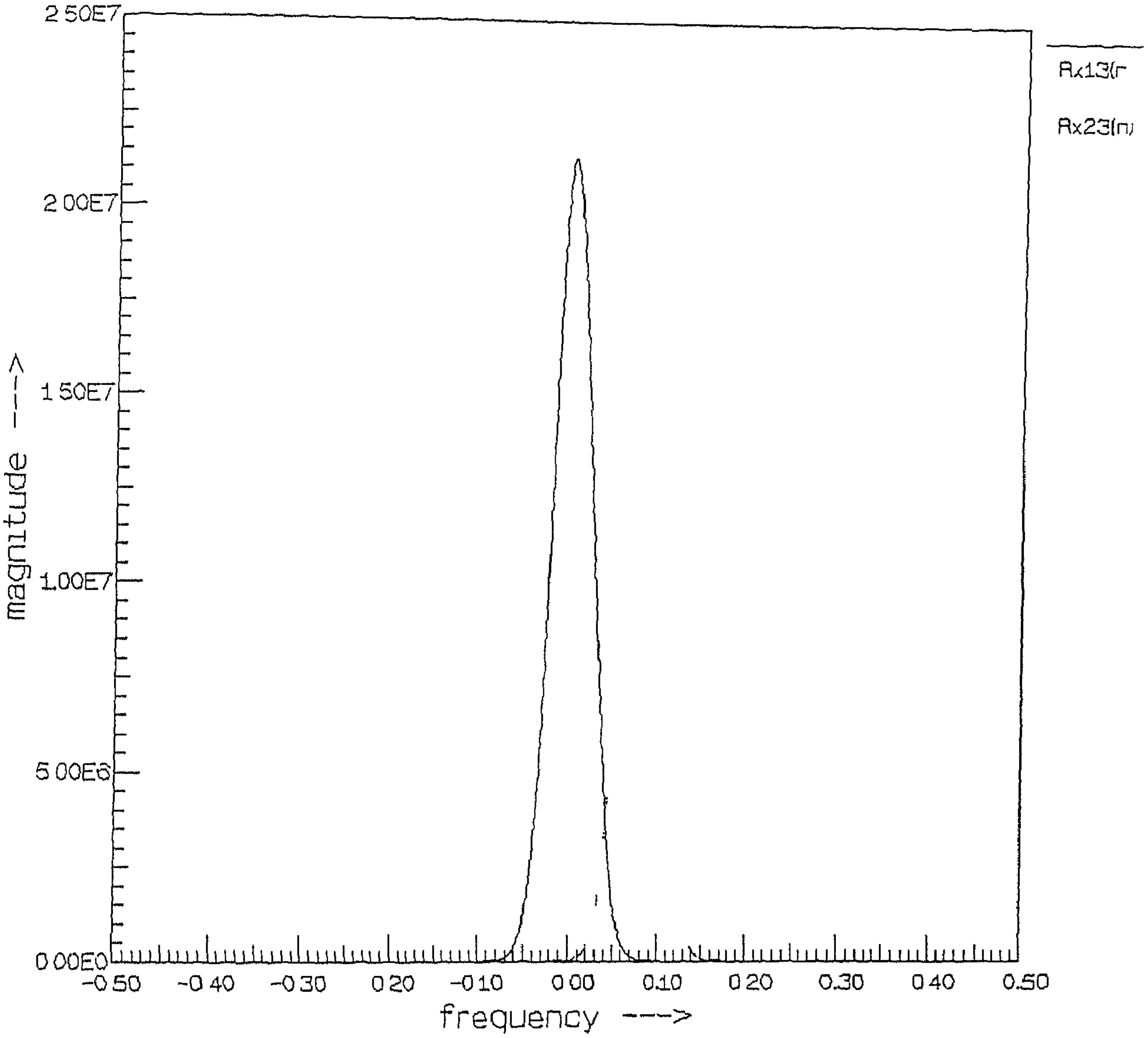


Fig 4 5 Estimation of alfa

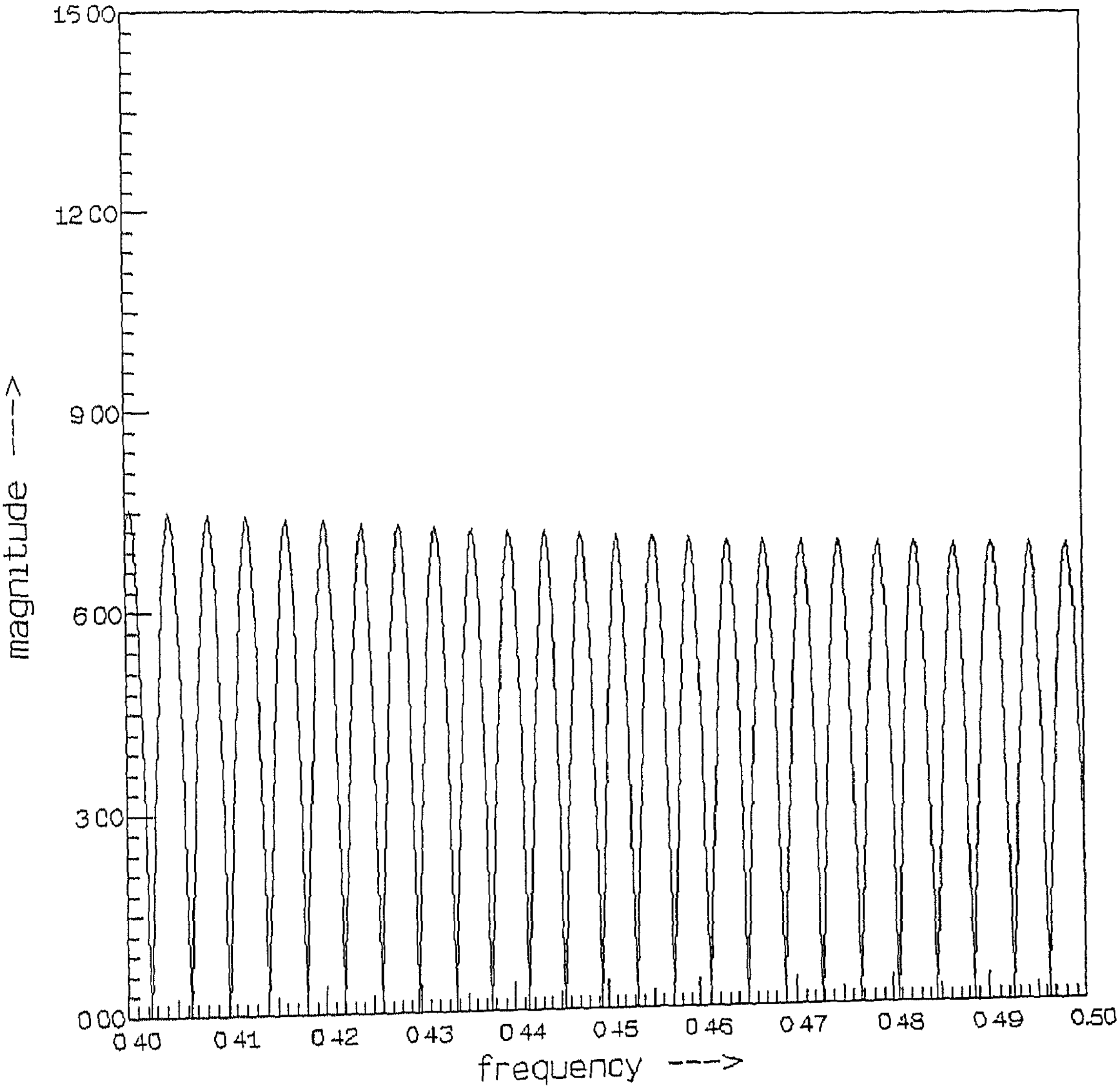


Fig 4 6 Est of theta 2 snr=30db

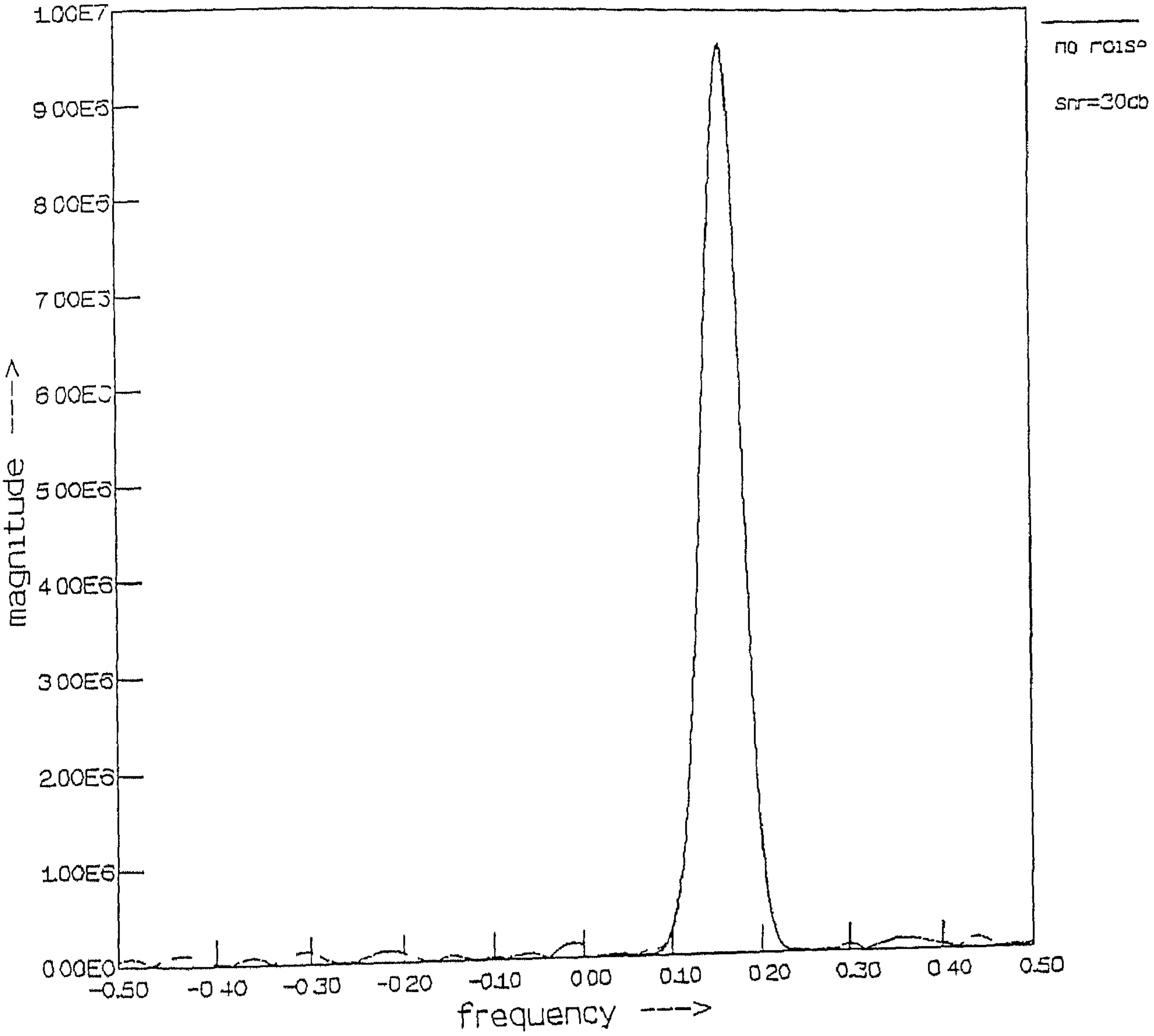


Fig 4 7 **Est of theta 2 snr=20db**

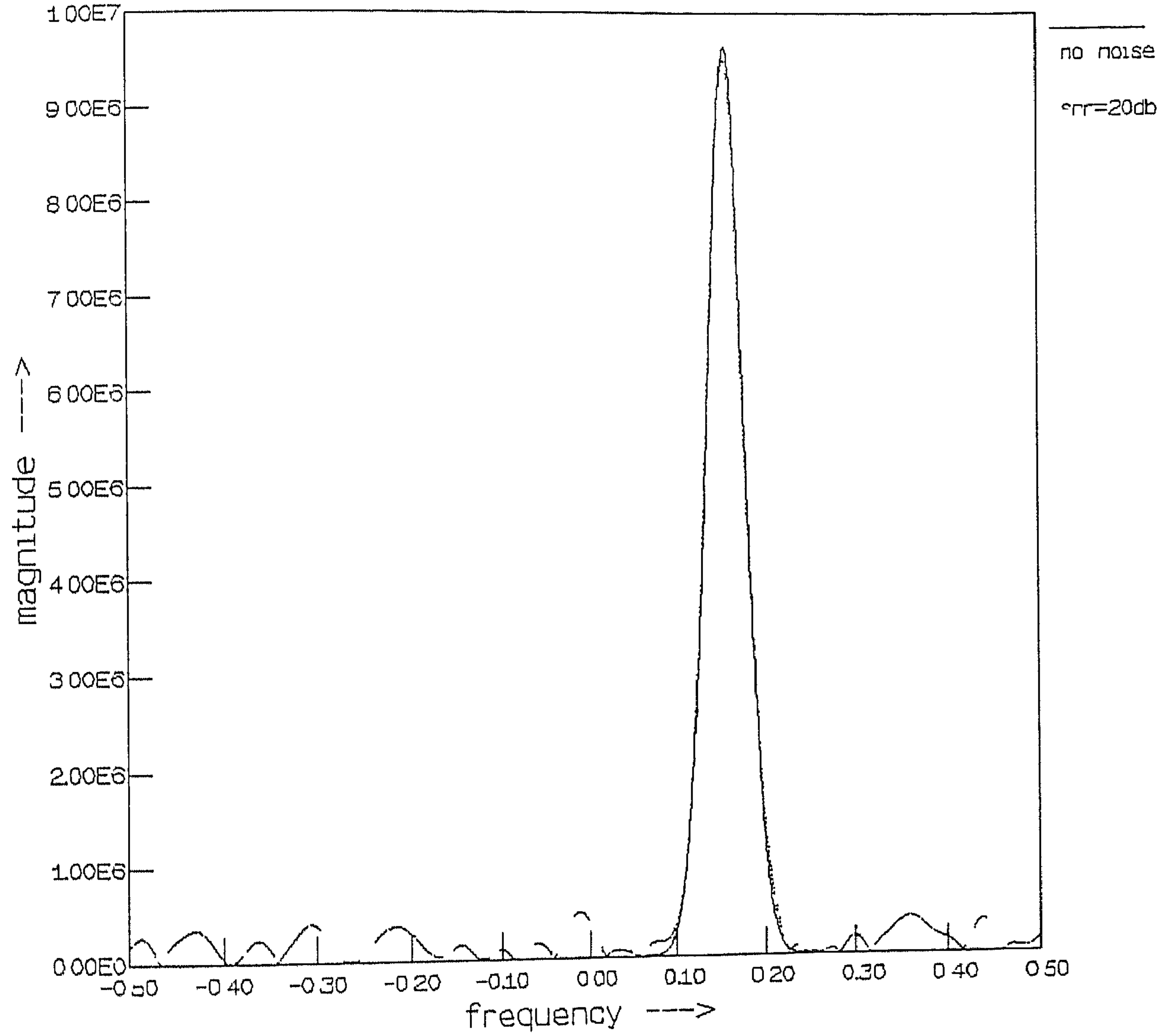


Fig 4 8 Est of theta 2 snr=10db

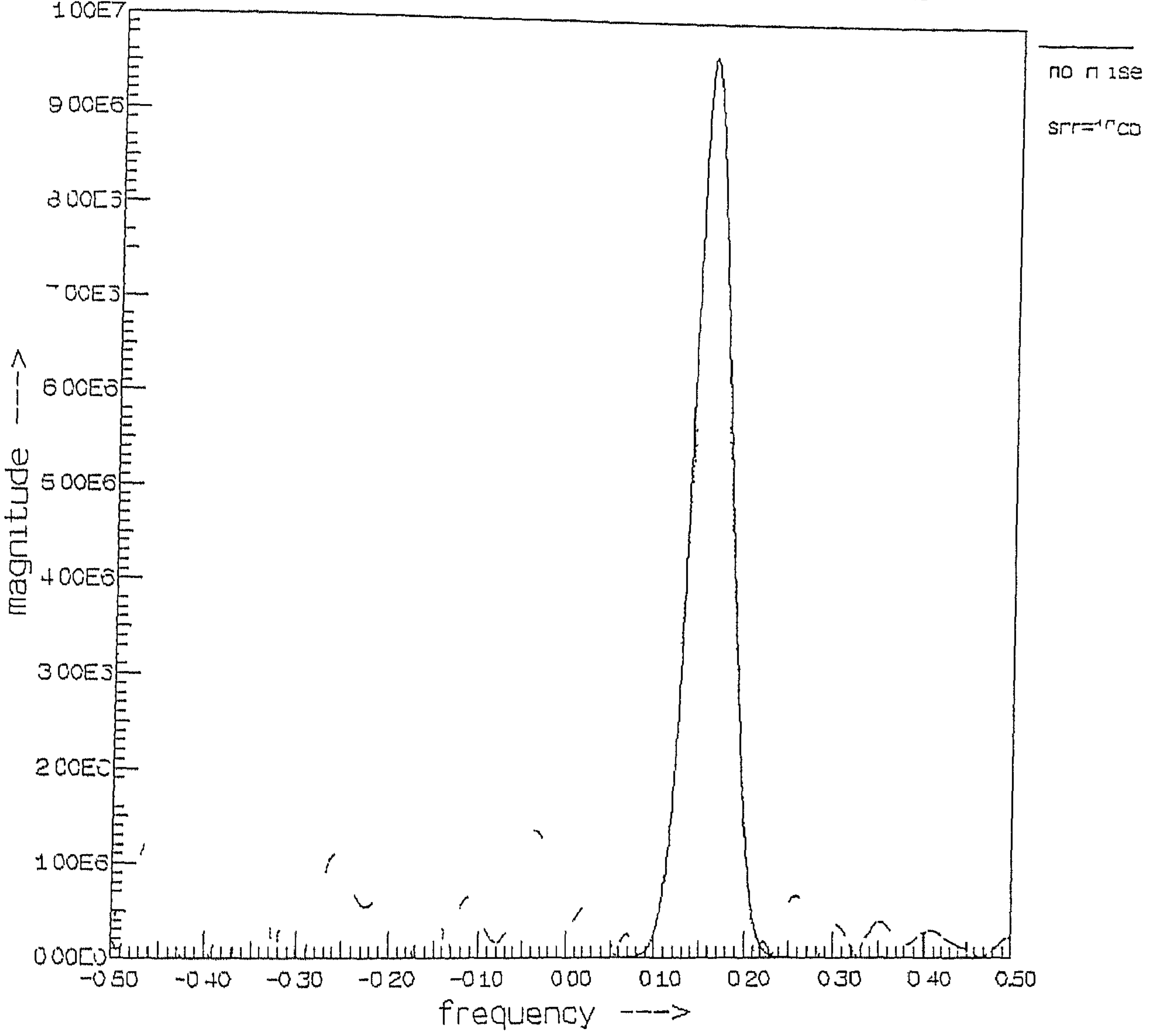


Fig 4 9 Estimation w/o noise

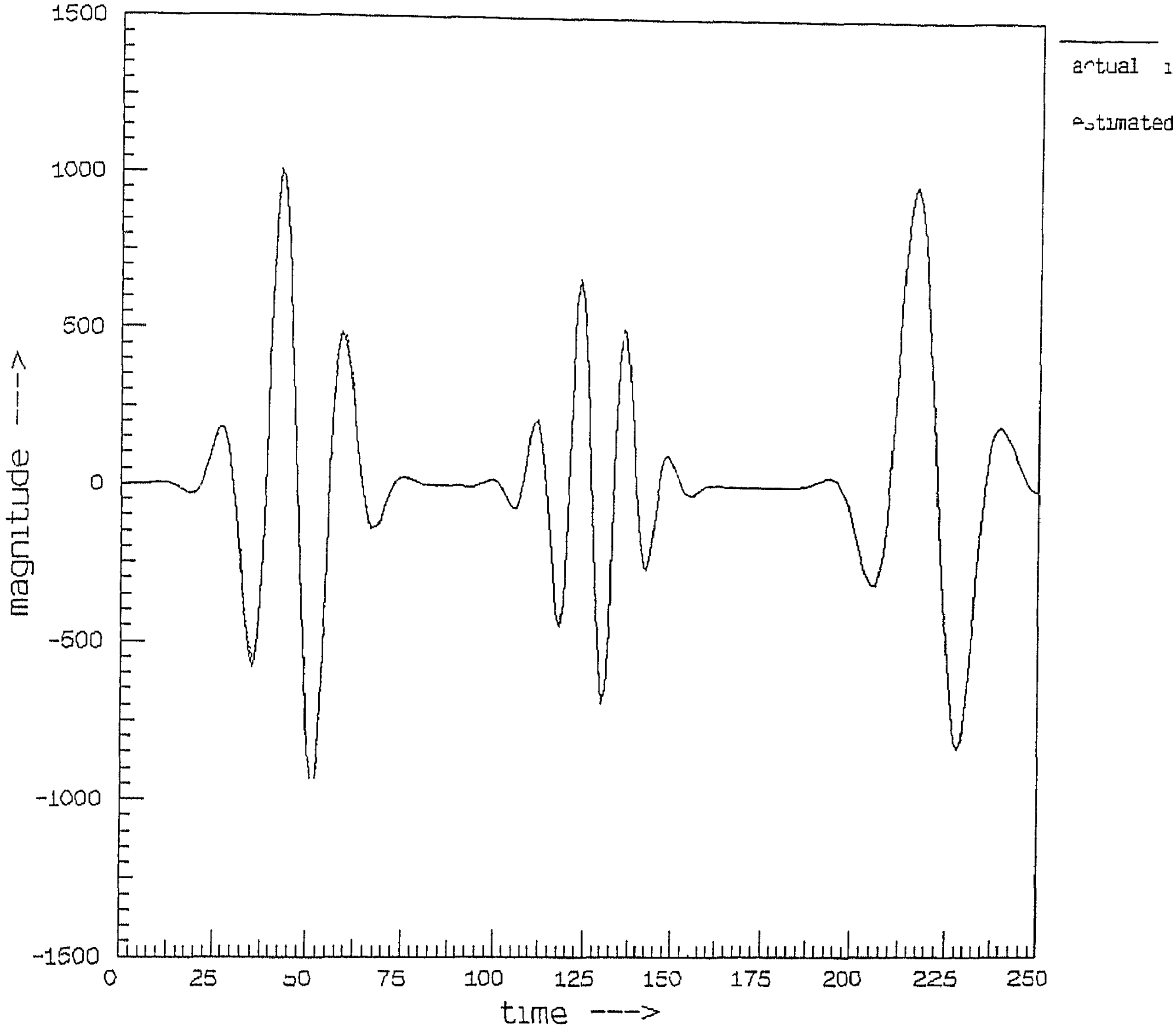
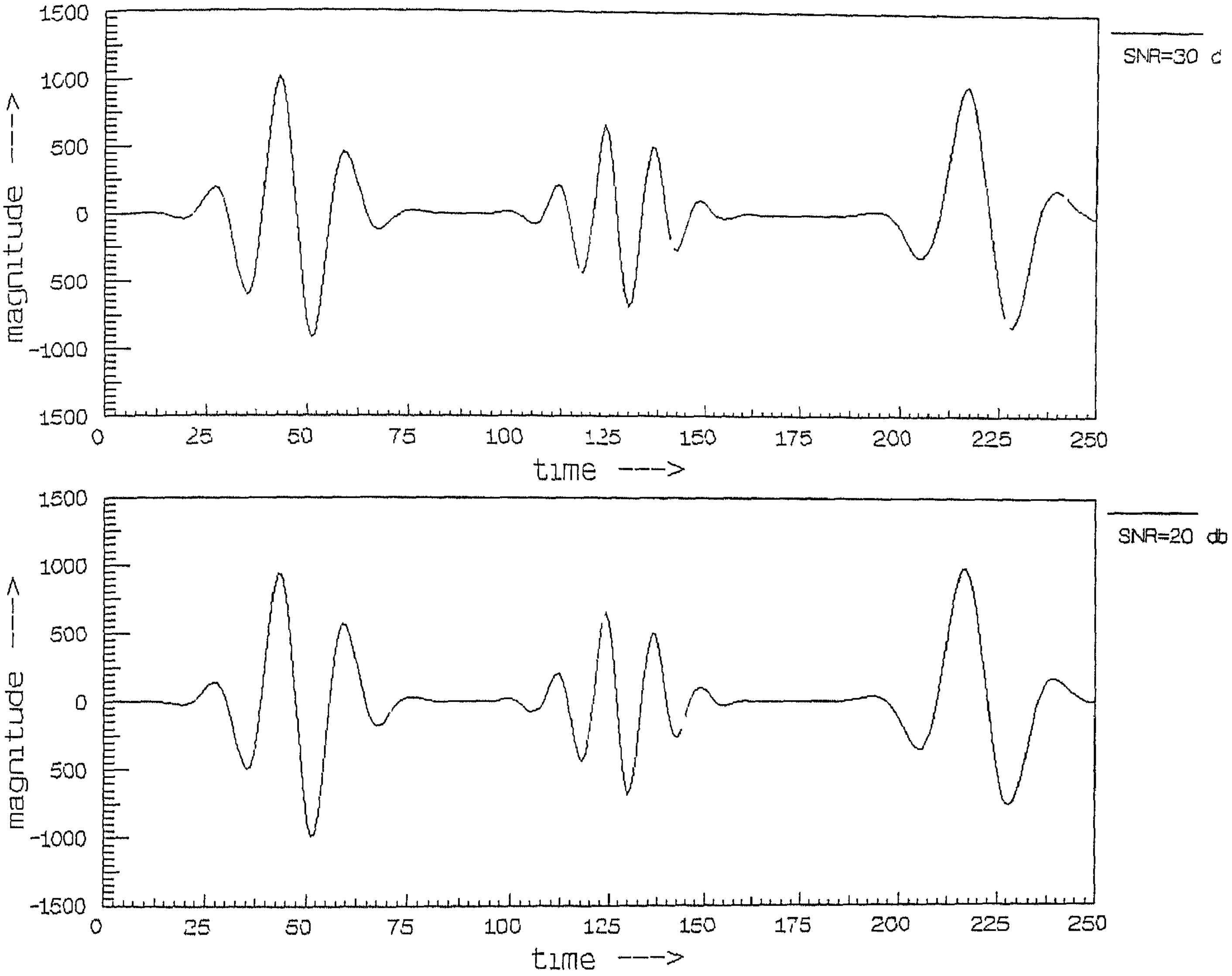


Fig 4 10 Estimation with noise



UsPlc

Fig 4 11 Complex ECG Signal

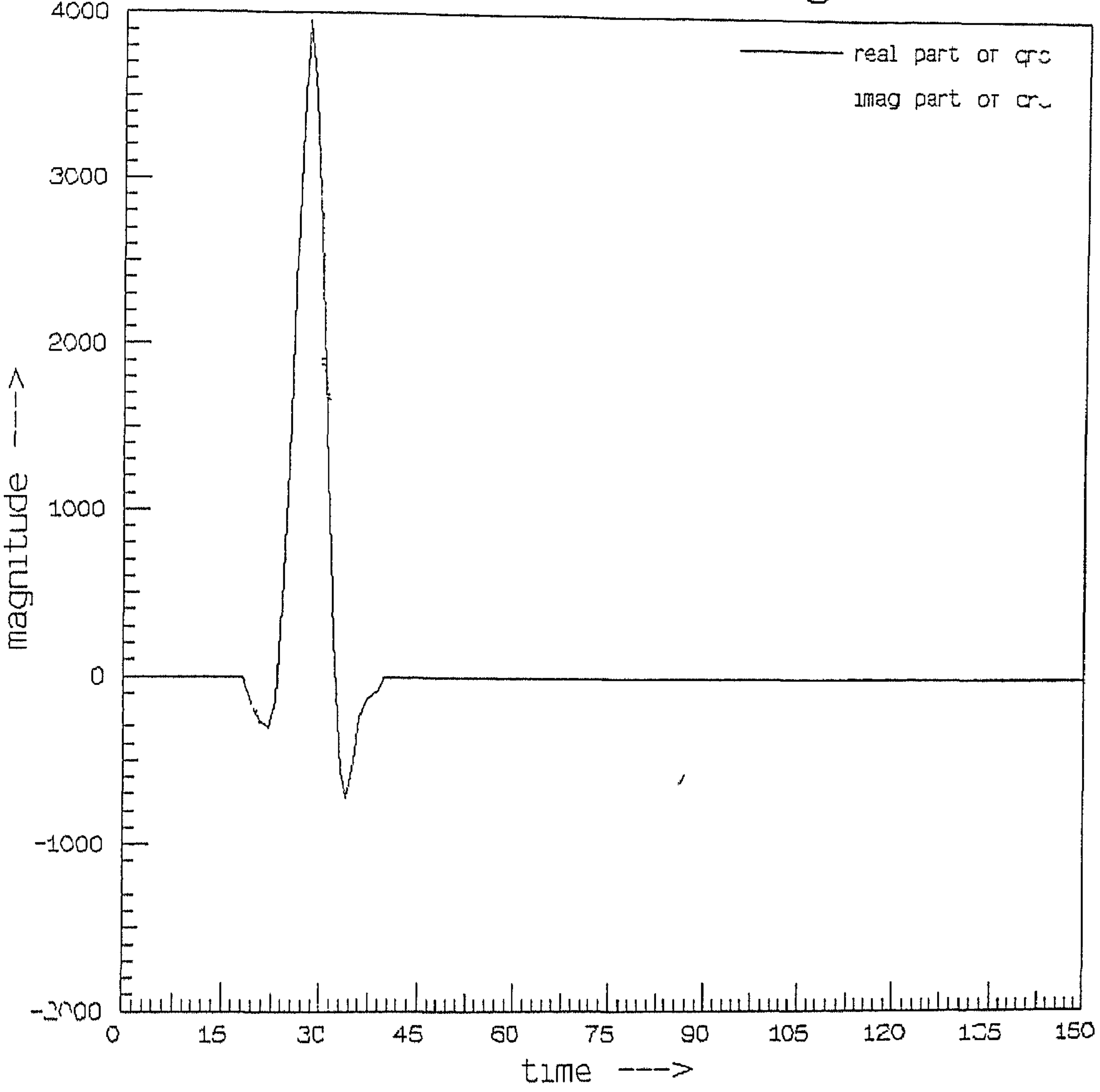


Fig 4 12 **Rx1(n) for QRS wave**

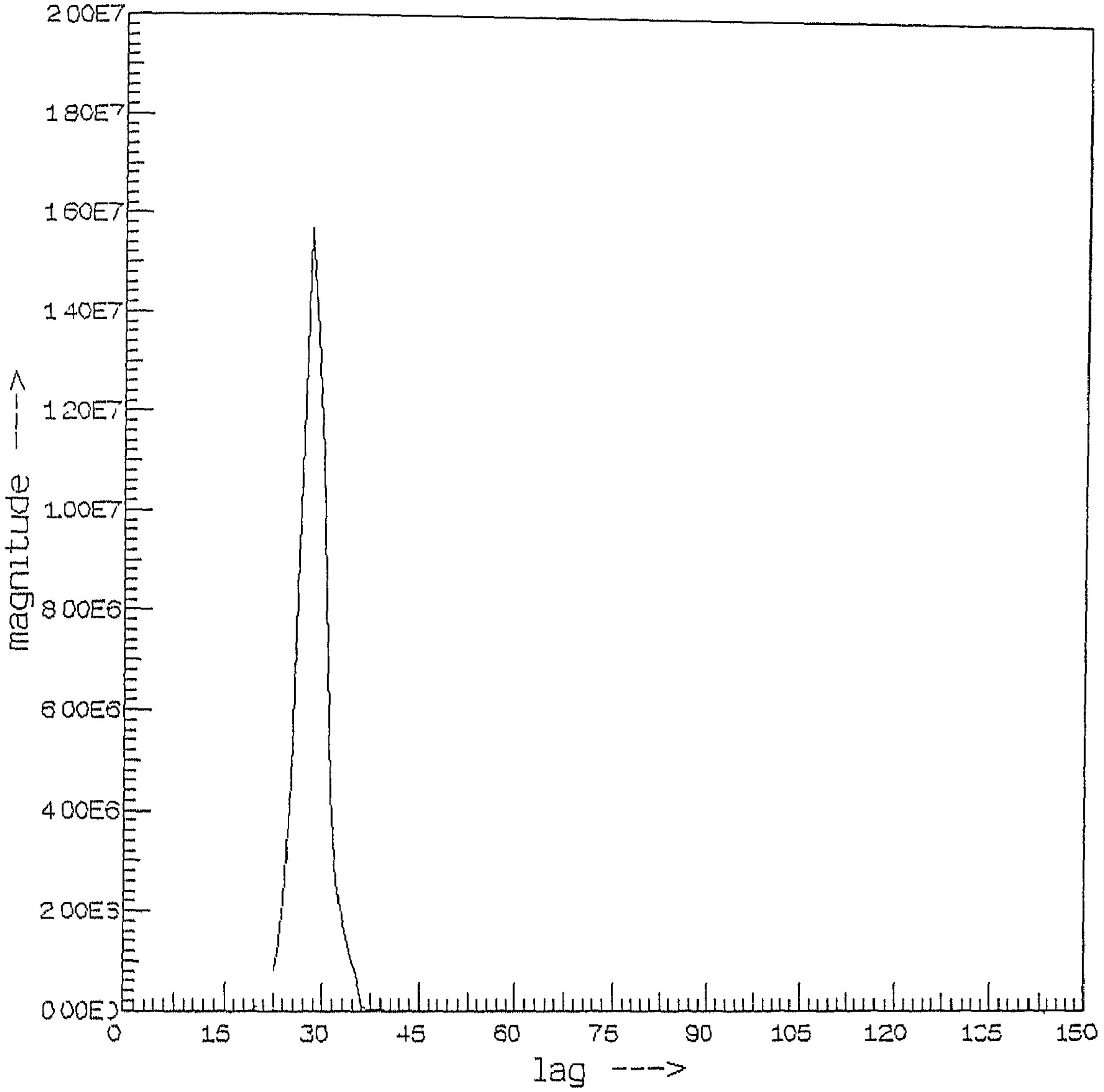


Fig 4 13 Est of theta:QRS wave

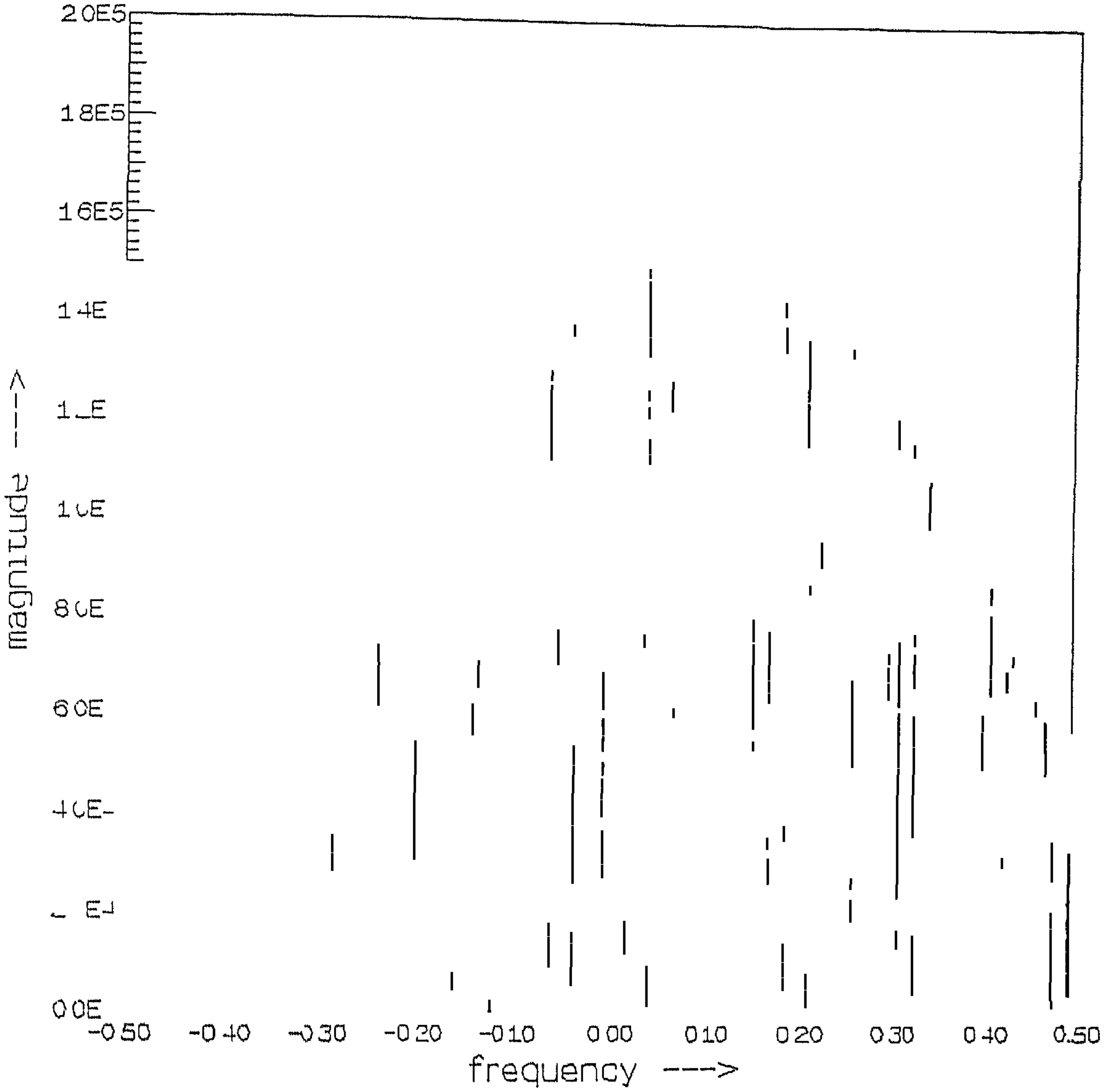
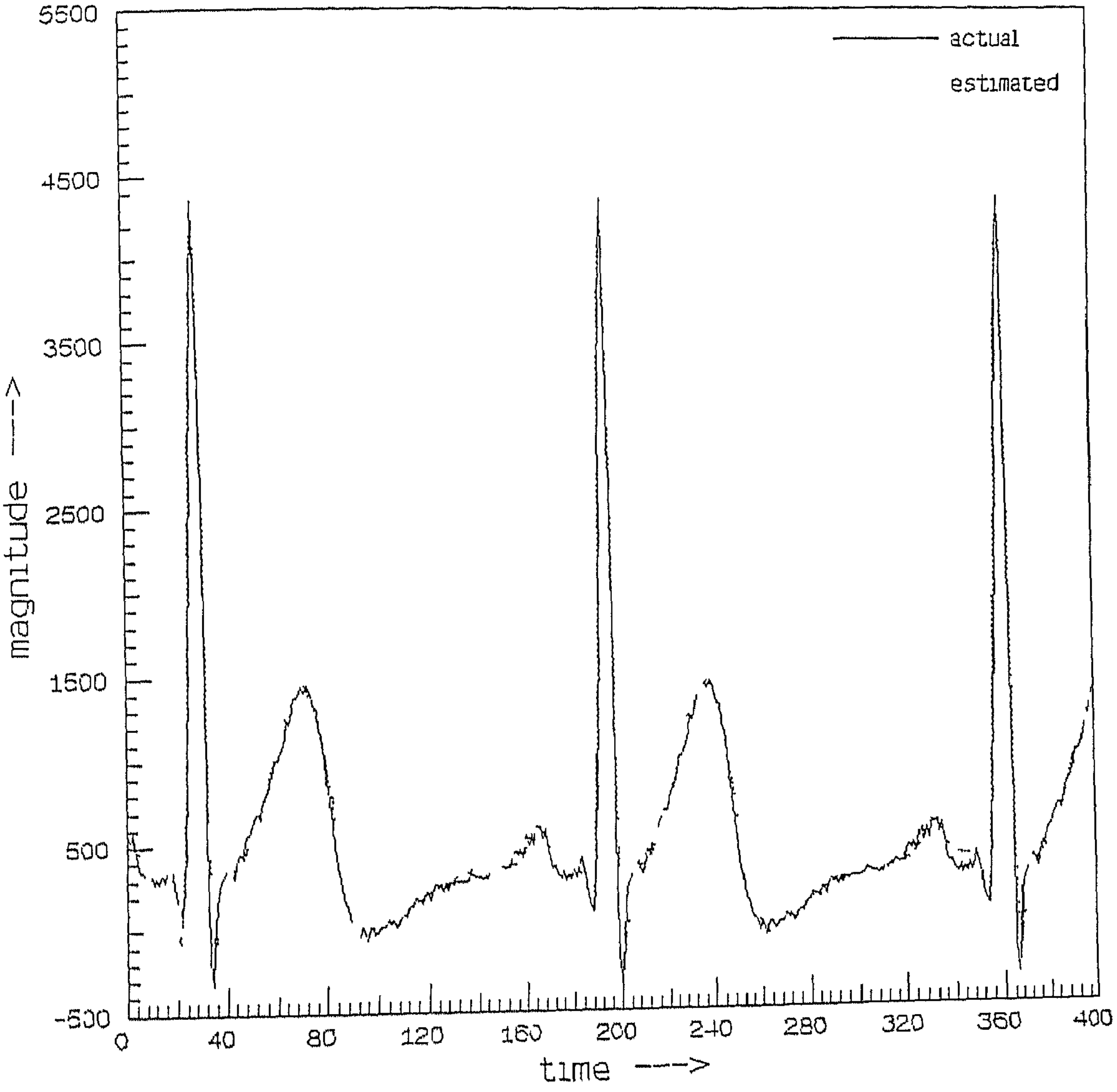


Fig 4 14 ECG Signal



CHAPTER 5

Conclusion

The method developed is quiet robust and accurate It requires that the signal being modelled should have certain well defined burst type features well demarcated in time The signals need not be periodic for periodicity can be imparted to the signal being modelled by considering small segments of the signal The method works quiet well in the presence of noise The model can be used for signals exhibiting sharp random bursts

In case of the ECG signal it provides a means to represent the ECG signal with a lesser amount of parameters It will be worthwhile to study how these parameters calculated in the model are related to the diagnosis of ailments if any The parameters afford a clear insight into irregularity in the action of any part of the heart, as far as timing information is concerned The irregularity in the execution of one cycle of the activity of the heart is also evident from the parameters However it requires a detailed study to see if these parameters can be directly connected to the diagnosis of other diseases of the heart

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